

## Reducing wheel wear of a motorised metro car on a curved track with a small curve radius

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A structural solution was proposed to enable passive control of the wheelsets of a motorised metro car on curved track with a small curve radius. This effect was achieved by varying the stiffness of the guidance of the wheelsets in the horizontal plane of the frame of each bogie on which the wagon body rests. This ensured the required critical speed of the metro wagon when moving on straight track, and allowed for reduced mechanical wheel wear and better steering of the bogies when moving on curved track with a small radius of curvature. Parameters specific to the design of modern metro cars were taken into account. Current knowledge of non-linear lateral dynamics of rail vehicles and the specialist computer programme Vi-Rail were used to perform the calculations involving simulation models.

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## 1. Introduction

Metro wagons are designed for operation in urban areas on lines made up of straight and curved track sections. Trains made up of such wagons run on straight track at operating speeds of 90 km/h. On curved track these speeds are lower and are determined by rail vehicle safety regulations [12]. Tracks with small curve radii affect the accelerated wear of the wheel and rail running surfaces, especially in the case of high stiffness of the wheelset guidance in the bogie frame. Satisfactory wheel wear in curved track with a small curve radius is only possible if the classic wheelsets are guided flexible in the bogie frame. In the case of rigid guidance, the problem of excessive wheel wear can be alleviated by forced steering of the wheelsets. Passive steering consists of an additional mechanical structure located outside the bogie frame. These may be lever mechanical systems in the form of rigid arms connecting opposite bearing housings of two wheelsets [6] or lever mechanical systems, controlling only a single axle in the bogie [13]. Another solution is lightweight sub-frames rigidly connected to the wheelsets and sprung between each other [6]. In

the case of active control in curved track, the rotation of the wheelsets relative to the bogie frame is forced in the horizontal plane by electro-mechanical actuators [2]. Such solutions require continuous measurement of the transverse displacements and angles of attack of the wheelsets on the rails, as well as the forces generated by the actuators on the primary and secondary vehicle suspension [9]. The authors of the article [11] proposed a solution to enable passive control of the wheelsets of a metro car with trailing two-axle bogies without the need for additional mechanical systems. The following article presents the results of a simulation study of motorised metro railcars with individual wheelset drive using asynchronous electric motors with a single-sided mechanical gearbox. Such a drive system increases the stiffness of the guidance of the wheelsets in the bogie frame, and thus adversely affects the wear numbers on the rolling surfaces of the wheels when moving on a curved track with a small curve radius. This paper analyses the results of simulation tests performed on a curved track with a curve radius of 300 m. The current knowledge of rail vehicle dynamics and the specialised computer programme Vi-Rail [14] were used to build a simula-

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tion model of a metro car equipped with powered, two-axle H-frame bogies without a swing bolster. The effect of passive control of the wheelsets of a motorised metro wagon on a curved track with a small curve radius was achieved by varying the stiffness of the guidance of the wheelsets in the horizontal plane of the frame of each bogie. This provided the wagon with the required non-linear critical speed on straight track and made it possible to significantly reduce mechanical wear on the wheels and improve the controllability of the metro wagon bogies on curved track.

## 2. Object of the research

The object of the research was a model of a metro car with two-axle powered bogies, whose design and parameters corresponded to those of real cars operated, for example, by the Warsaw Metro [5].

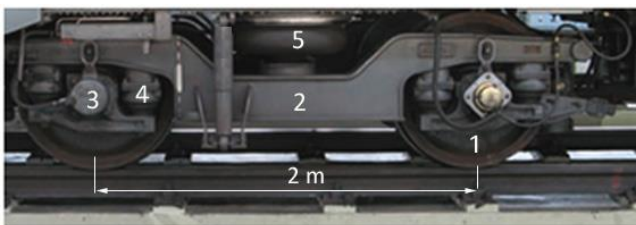


Fig. 1. View of the bogie under the metro car body [5]

The suspension elements and structure of the driving bogie are shown in Fig. 1 and Fig. 2. The primary suspension of the wagon consisted of metal-rubber springs 4 placed between the housing 3 of the axle bearings of the wheelsets and the bogie frame 2 – Fig. 1. The secondary suspension consisted of two pairs of gas springs 5, seated on the H-frame of a two-axle bogie without a swing bolster – Fig. 2.

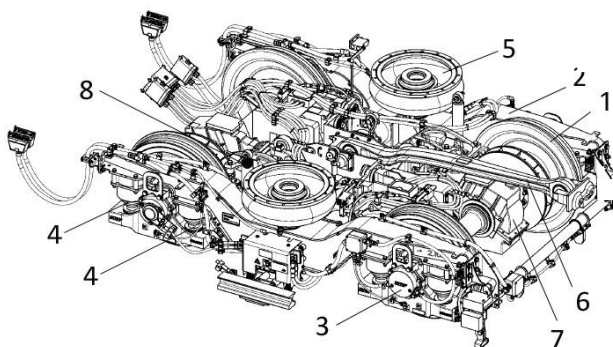


Fig. 2. Sketch of a metro car motor bogie [2]

The springs 5 provided support for the car body. Asynchronous motors 8 were used to drive the wagon bogies – Fig. 2. The driving torque was transmitted to the axle of the selected wheelset via a single-sided pinion gear 7, while a traction rod 6 was used to

transmit the longitudinal forces from the bogies to the wagon body [3]. The elements of the structure of a metro car model are shown in Fig. 3.

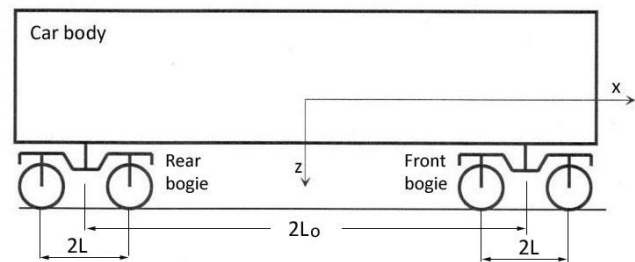


Fig. 3. Diagram of the structure of a metro car model

## 3. Research problem

The magnitude and speed of wheel wear are most influenced by curved tracks with small curve radii and the high stiffness of the wheelset guide in the bogie frame. Advanced wear of the wheel profile can be easily detected visually using a suitable template, as shown in Fig. 4.



Fig. 4. Visualisation of wear on the wheel running surface [8]

The research problem was reduced to finding the simplest possible design solution to reduce wheel wear and improve the steerability of the rigid bogies of a metro car on curved track with a small curve radius. According to [4], a minimum curved track curve radius of 300 m was assumed. A metro wagon with two-axle bogies was considered, in which the stiffness of the wheelset guidance in the horizontal plane of the frame of each bogie was varied. The Vi-Rail software was used for the dynamic tests [14].

## 4. Simulation model

The phenomenological model of the motor bogie is shown in Fig. 5. The directional stiffnesses of the primary suspension associated with a given axle bearing housing were described as  $0.5k_x$ ;  $0.5k_y$ ;  $0.5k_z$ . This meant that the guiding stiffnesses of a single wheelset in the horizontal transverse plane were equal to  $k_x$ ,  $k_y$ . The longitudinal stiffness of the traction rod

was denoted by  $kx_{tr}$ . The directional suspension stiffnesses of the secondary suspension were described by the symbols  $KX$ ;  $KY$ ;  $KZ$ . The stiffness values of the primary suspension of the actual Metropolis underground car were taken as nominal, according to [5] and [11]. This means that  $kx = ky = 6.600$  MN/m,  $kz = 1.880$  MN/m. In the simulation calculations, the stiffness values of  $kx$  and  $ky$  were varied from 4 MN/m to 14 MN/m. The accumulated values of the other metro wagon parameters in Table 1 were also treated as nominal and did not change during the subsequent simulations. A pair of indices  $(i, j)$  was used to identify the selected wheel in the metro car model structure. The index  $i = 1, 2, 3, 4$  indicated the wheelset number, while  $j = 1, 2$  was the wheel number in the wheelset. According to this principle, the position of all wheels in the four-axle metro car model was determined – Fig. 6.

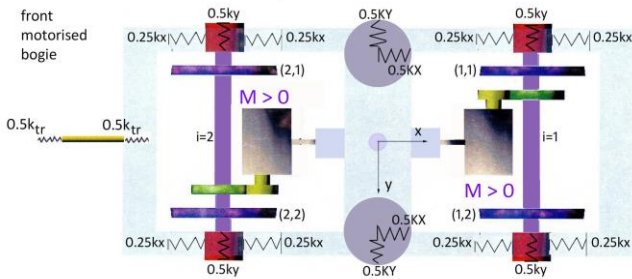


Fig. 5 Phenomenological model of the motor bogie [14]



Fig. 6. Identification of wheel positions in a four-axle metro car model

The simulation model of the metro car was extended by including a mathematical model of a PWM-controlled asynchronous motor [10]. The description of the wheel-rail contact took into account the non-linear S1002 outlines of the wheels and the 60E1 rails with a gauge of 1.435 m and a slope of 0.025 rad. The track parameters were adopted in accordance with the requirements contained in [4]. The stiffness of the wheelset guidance in the horizontal plane of the frame of each of the two bogies of the wagon was varied from 4 MN/m to 14 MN/m. The symmetrical configuration was characterised by equal values of the wheelset guiding stiffnesses. The values of  $kx = ky = 4$  MN/m gave the bogies characteristics close to those of flexible bogies, while the stiffnesses of  $kx = ky = 14$  MN/m gave the characteristics of rigid bogies. In the case of the asymmetric configuration of the stiff-

ness of the wheelset guidance in the bogie frame, the values of these stiffnesses differed within the previously assumed range. At the same time, bogies with an asymmetric configuration of the wheelset guiding stiffness were positioned under the body in such a way as to obtain an arrangement providing the model metro car with the same dynamic properties irrespective of the direction of movement. The letters **a** and **b** were used to symbolically describe the axle-guiding stiffness configuration of the wheelsets in the bogie frame – Fig. 7. The axle-guiding stiffness configuration of a bogie wagon with a symmetrical configuration was described symbolically as **a – a** – **a – a**. In this case, the letter **a** identified equal guiding stiffness values for all wheelsets.

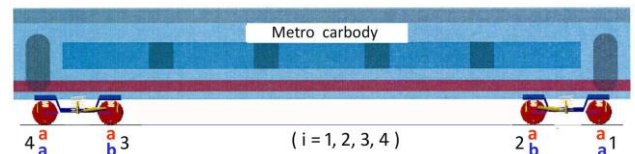


Fig. 7. Symbolic description of the axlebox guidance stiffness configuration in a simulation model of a metro car

The arrangement of the wheelset guiding stiffnesses in a model wagon with bogies of asymmetrical configuration was described as follows: **a – b** – **b – a**. The letter **a** identified equal values of the wheelset guiding stiffnesses numbered 1 and 4, while the letter **b** indicated equal values of the wheelset guiding stiffnesses numbered 2 and 3.

Table 1. Parameters of a simulation model of a metro car

Parameter	Unit	Value
$2L_0$	m	12.6
$2L$	m	2
$m$	kg	29600
$kx$	MN/m	(3, 4, 6, 8, 10, 12, 14)
$ky$	MN/m	(3, 4, 6, 8, 10, 12, 14)
$kz$	MN/m	1.88
$kx_{tr}$	MN/m	10
$0.5KX$ ; $0.5KY$	MN/m	0.45 MN/m
$KZ$	MN/m	0.690
$0.5cx$ ; $0.5cy$ ; $0.5cz$	kNs/m	3
$0.5CX$ ; $0.5CY$ ; $0.5CZ$	kNs/m	2
$Q$	kN	36.3
$r$	m	0.43
$\mu$	–	0.36

The designations  $2L_0$ ,  $2L$  given in Table 1 referred to the wagon base and bogie base, respectively, while  $cx$ ;  $cy$ ;  $cz$  and  $CX$ ;  $CY$ ;  $CZ$  specified the directional damping coefficients in the primary and secondary suspension of the wagon. The nominal rolling radius of the running wheel was denoted by the symbol  $r$ . The coefficient of friction between wheel and rail was

described by the symbol  $\mu$ , while the allowable static pressure of a single wheel on the rail was designated as  $Q$ . An energy model (the so-called "T-gamma model") based on the assumption that the amount of material lost is a function of the energy dissipated in the area of contact between a given wheel and the rail was used to assess wear on the rolling surface of the wheels [14]. For this purpose, the values of the wear numbers  $w(i, j)$  were calculated. These wear numbers were defined as the sum of the products of the tangential forces and the sliding slips in the wheel-rail contact area:

$$w(i, j) = |T_x(i, j) \cdot \gamma_x(i, j)| + |T_y(i, j) \cdot \gamma_y(i, j)| \quad (1)$$

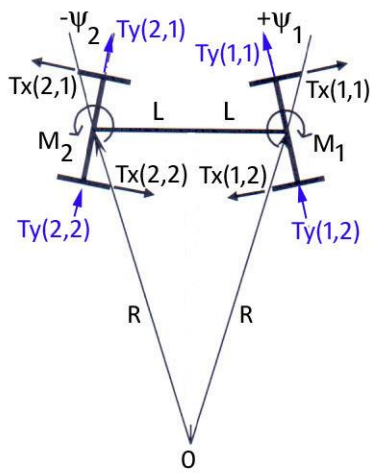


Fig. 8. Signs of angles of attack and tangential forces acting on bogie wheels on curved track

Physically, the wear number was interpreted as the amount of energy dissipated over the path of one metre by the longitudinal  $T_x(i, j)$  and lateral  $T_y(i, j)$  tangential forces acting in the contact area between the selected wheel and the rail – Fig. 8. In formula (1), the symbols  $\gamma_x(i, j)$  and  $\gamma_y(i, j)$  denote, respectively, the relative longitudinal and transverse slip of a given wheel in the system associated with the track. The steerability  $\theta_k$  of a bogie on a curved track is defined as follows:

$$\theta_k = \frac{1}{2} (\Psi_i + \Psi_{i+1}) \quad (2)$$

The symbols  $\Psi_i$  and  $\Psi_{i+1}$  denote the angles of attack of the wheelsets on the rails. The index  $k = 1, 2$  determines the number of the bogie in the wagon. According to the above formula, perfect bogie steerability occurs when  $\theta_k = 0$ . The moments acting on a given wheelset, exerted by a pair of longitudinal tangential forces  $T_x(i, j)$ , are denoted by the symbols  $M_i$ .

### 5. Calculation results

The metro car model started running on straight track with an acceleration of  $1.2 \text{ m/s}^2$ , then traversed the transition curve and entered a full curve of curved track with a radius of 300 m having a cant of 150 mm. The results of the calculations were recorded on the full curve as the wagon model achieved and maintained the target speed of 20 m/s, taking into account the fundamental resistance to motion [1]. A graph of the speed and driving torque transmitted to the axle of a single wheelset in the driving bogie is shown in Fig. 9.

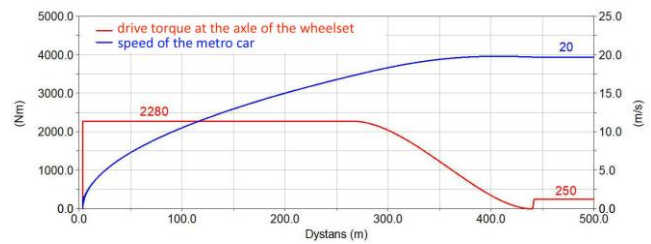


Fig. 9. Model metro wagon speed and driving torque at the axle of the wheelset as a function of distance [14]

In each calculation step, a system of differential-algebraic equations was solved, created on the basis of information about the coordinates of the centres of gravity of individual model members, their masses and mass moments of inertia. To determine the forces acting in the areas of contact between the wheels and the rails, the FASTSIM procedure was used [7]. A table of contact parameters [14] was used to determine the tangential forces, between wheels and rails. In each calculation step, the difference of the actual rolling radii of the wheels in the wheelsets, the wheel-rail contact angles and the half-axes of the contact ellipse of the respective wheel-rail contact were read from this table.

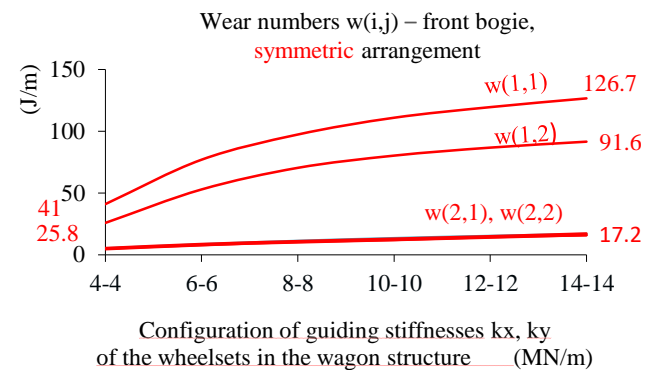


Fig. 10. Wheel wear numbers for the front bogie with symmetrical wheelset guide stiffness configuration

The numerical values of wheel wear numbers  $w(i, j)$  of a metro car model with bogies with symmet-

rical configurations of 4 – 4\_\_4 – 4 , 6 – 6\_\_6 – 6, 8 – 8\_\_8 – 8 , 10 – 10\_\_10 – 10 , 12 – 12\_\_12 – 12, 14 – 14\_\_14 – 14 wheelset guiding stiffness, are shown in Fig. 10 and Fig. 11. On the basis of the test results published in [11], it was assumed that in the asymmetric configuration the guiding stiffnesses of the first and fourth wheelsets in the wagon would be constant and equal to 4 MN/m. However, the values of the guidance stiffness of wheelsets numbered 2 and 3 were varied, ranging from 6 MN/m to 14 MN/m.

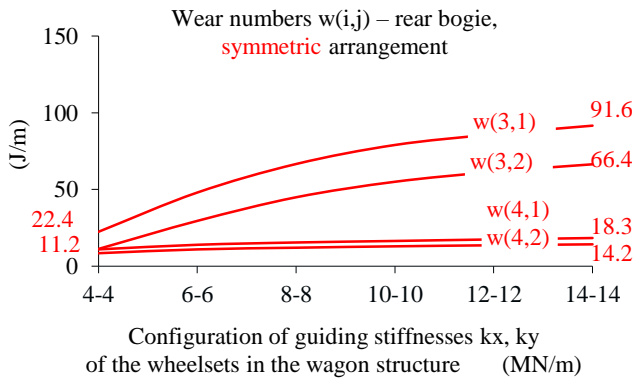


Fig. 11. Rear bogie wheel wear numbers with symmetrical wheelset guide stiffness configuration

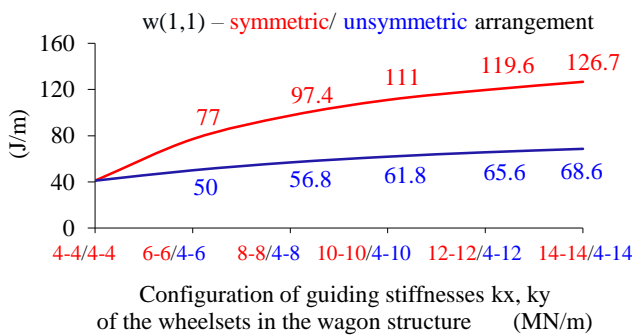


Fig. 12. Comparison of the values of the wear numbers for the left wheel in the symmetrical and asymmetrical stiffness configuration of wheelset number 1

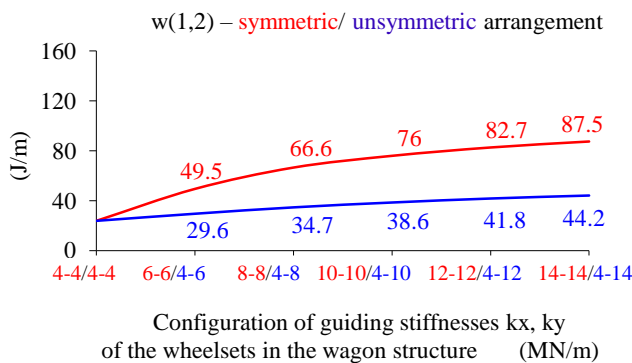


Fig. 13. Comparison of the values of the wear numbers for the right wheel in the symmetrical and asymmetrical wheelset guide stiffness configuration number 1

The following five configurations of wheelset guiding stiffness in the wagon model were thus obtained: 4 – 6\_\_6 – 4 , 4 – 8\_\_8 – 4, 4 – 10\_\_10 – 4 , 4 – 12\_\_12 – 4 , 4 – 14\_\_14 – 4. Figure 12 and Figure 13 graphically compare the values of the wheel wear indices  $w(1,1)$  and  $w(1,2)$  in the leading axle set of the front bogie as a function of symmetric and asymmetric configurations of the leading axle set of this bogie. In order to establish the relationship between the values of the wheel wear indices  $w_{asymm}(i,j)$  in the asymmetric wheelset guiding stiffness configuration and the wear indices  $w_{symm}(i,j)$  in the symmetric configuration, a quality assessment factor  $p(i,j)$  of the solution with asymmetric wheelset guiding stiffness in the bogie frame was introduced:

$$p(i,j) = \frac{w_{symm}(i,j) - w_{asymm}(i,j)}{w_{symm}(i,j)} 100\% \quad (2)$$

This coefficient was expressed as a percentage. In this way, the level of reduction in the values of wheel wear numbers in the asymmetrical configuration of wheelset guidance stiffness in the bogies was assessed compared with the symmetrical configuration. The results are shown in Fig. 14 and Fig. 15.

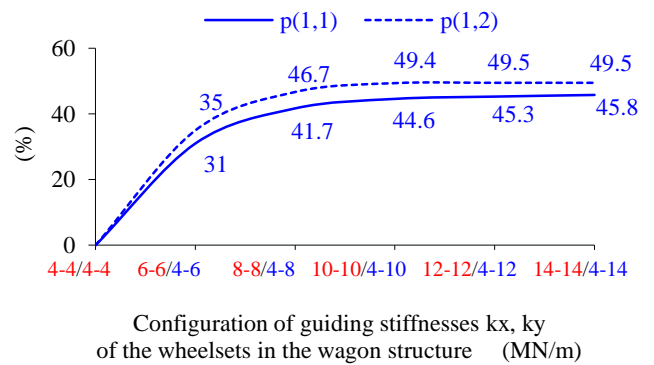


Fig. 14. Values of the quality factor for the solution with asymmetry of the stiffness of the wheelset guide for the wheelset number 1

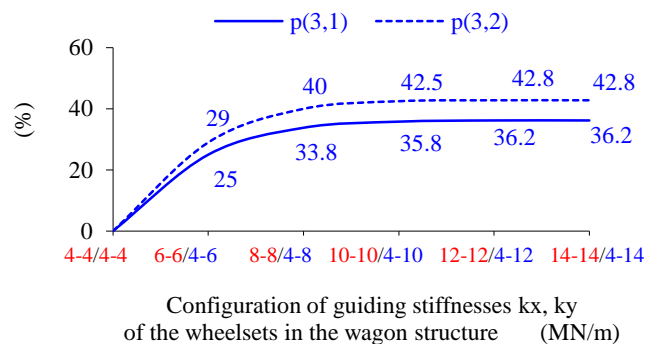


Fig. 15. Values of the quality factor for the solution with asymmetry of the stiffness of the wheelset guide for the wheelset number 3

6. Discussion

The subject of the simulation study and analysis was a motorised metro car model with two-axle bogies and individual wheelset drive. Metro trains travel on straight track with a maximum operating speed of 90 km/h. In this paper, it is assumed that the non-linear critical speed of the metro car model should be at least 20% higher than the operating speed. According to the calculation scenario adopted, the wheelset guiding stiffnesses in the frame of each bogie were assigned values ranging from 4 MN/m to 14 MN/m, investigating symmetrical and asymmetrical configurations. The numerical values of the non-linear critical speed of the metro wagon model as a function of the asymmetrical configurations of the wheelset guidance stiffness in the frame of each bogie are shown in Fig. 16. Each of the asymmetrical configurations taken into account provided a correspondingly high non-linear critical speed and thus safety in straight-track movement of the metro wagon model.

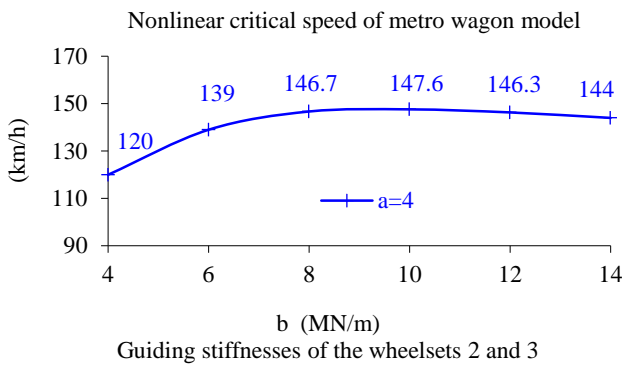


Fig. 16. Non-linear critical speed of a metro car model with bogies with asymmetric wheelset guidance stiffness configuration

In the symmetrical 14 – 14\_\_14 – 14 configuration of guiding stiffness, the values of the wear numbers of the wheels in the leading wheelsets increased dramatically compared to the symmetrical 4 – 4\_\_4 – 4 configuration. For the leading wheelsets of the front and rear bogie, there was a more than four times increase in the values of the wear numbers. This is shown in Fig. 10 and Fig. 11. The wheel wear numbers in the trailing wheelsets were many times lower compared to the leading wheelsets and practically remained at a uniform level over the entire range of considered leading wheelset stiffness values. In the category of minimising wheel and rail wear, the obvious choice would be to leave the guidance stiffness of all four wheelsets at 4 MN/m. However, in the case of motorised bogies, the propulsion system significantly increases the guidance stiffness of the wheelsets giving the bogies of a metro car the characteristics of

rigid bogies. Thus, the question of how to reduce the high values of wheel-wear numbers occurring in the case of rigid motor bogies was relevant. The introduction of an asymmetrical configuration of the wheelset guide stiffness in the frame of each bogie offered this possibility. The expected effect is illustrated in Fig. 12 and Fig. 13. The asymmetrical configuration of the wheelset guidance in the bogie frame made it possible to keep the guidance stiffness of one of the wheelsets in each bogie as high as possible. In the metro car model structure, this was the case for wheelsets numbered 2 and 3. The relationship between the wear numbers of the wheels in the asymmetrical configuration and the wear numbers in the symmetrical configuration of the axle guiding stiffness of the wheelsets in the frame of each bogie is shown in Fig. 14 and Fig. 15. By comparing two bogie car model layouts with symmetrical 14 – 14\_\_14 – 14 and asymmetrical 4 – 14\_\_14 – 4 configurations of bogie wheelset guidance stiffness, it was shown that the asymmetrical 4 – 14\_\_14 – 4 configuration enabled the magnitude of wheel-wear numbers in the metro car model to be reduced by at least 40%. The bogie steerability and tangential force pair moments  $T_x(i, j)$  acting on a given wheelset were also determined in these two cases. The results are shown in Fig. 16 and Fig. 17. The bogie steerability and tangential force pair moments  $T_x(i, j)$  acting on a given wheelset were also determined in these two cases. The results are shown in Fig. 17 and Fig. 18. The smaller running angles of the wheelsets on the rails resulted in the front bogie adopting a position closer to the centre of the track and the rear bogie moving towards the outer rail in the asymmetrical configuration compared to the symmetrical configuration with rigid bogies – Fig. 18.



Fig. 17. Position of bogies of a metro car model with a symmetrical 14 – 14\_\_14 – 14 configuration of wheelset guidance stiffness on a full curve of track with a radius of 300 m

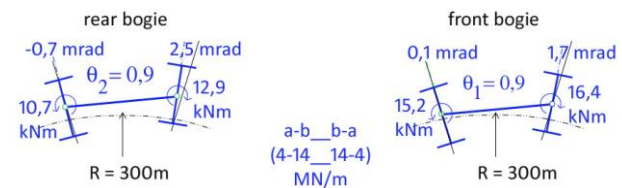


Fig. 18. Position of bogies of a model metro car with asymmetric 4 – 14\_\_14 – 4 configuration of the wheelset guidance stiffness on a full curve of the track with a radius of 300 m

This resulted in the expected near-radial alignment of the wheelsets on the full curve of the curved track. This alignment was also influenced by the larger magnitudes of the moments exerted on the wheelsets by the longitudinal tangential forces in the transverse tangential plane to the curved track compared to the symmetrical configuration. The numerical values of these moments are given in Fig. 18. The steerability of the front bogie with asymmetrical 4 – 14\_\_14 – 4 wheelset guidance stiffness was 2.9 times better and that of the rear bogie was 2.1 times better compared with that of the bogies in the symmetrical 14 – 14\_\_14 – 14 wheelset guidance stiffness configuration. In addition, for the asymmetrical 4 – 14\_\_14 – 4 configuration, the steerability of each bogie of the metro car model was the same.

## 7. Final conclusions

The dynamic properties of a simulation model of a motorised metro car travelling at an acceptable speed on a curved track with a curve radius of 300 m were analysed in terms of wheel wear and bogie steerability. This paper presents the results of a simulation study that confirmed the expectations of the possibility of reducing wheel wear of a metro car with powered bogies on curved track with a small curve radius. It was shown that it is possible to maintain a very high steering stiffness of one of the wheelsets in the bogie frame if the steering stiffness of the other wheelset is correspondingly low. It was determined that in the

wagon structure guidance stiffnesses of the order of 14 MN/m should be assigned to wheelsets numbered 2 and 3. This made it possible to use significantly lower guidance stiffnesses, at the level of 4 MN/m, for wheelsets numbered 1 and 4. In this way, the effect of passive control of the bogie wheelsets was achieved without the need for additional systems forcing radial alignment in the curved track. The proposed asymmetrical configuration of bogies in the structure of the 4 – 14\_\_14 – 4 metro wagon model made it possible to reduce the magnitude of wheel wear indicators in the leading wheelsets by at least 40% in comparison with the symmetrical 14 – 14\_\_14 – 14 configuration of rigid bogies. At the same time, better bogie steerability on the curved track under study was achieved in this comparison. The controllability of the front bogie with 4 – 14\_\_14 – 4 asymmetry of wheelset guidance stiffness was 2.9 times better and that of the rear bogie was 2.1 times better compared to the steerability of the bogies in a symmetrical 14 – 14\_\_14 – 14 configuration of wheelset guidance stiffness. The introduced asymmetry of the wheelset guidance stiffness in the frame of each driving bogie also allowed the metro car model to achieve a non-linear critical speed of 144 km/h on straight track. Thus, the safety requirements for straight-track movement were maintained with the full guarantee of the wagon moving at an operational speed of at least 90 km/h.

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