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Equivalent conicity as a parameter for assessing track condition

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Safety in rail transport depends directly on the technical condition of the track and the wheel–rail interface. Reliable assessment of infrastructure condition is therefore essential, particularly under increasing operational loads. In addition to conventional geometric measurements, parameters describing wheel–rail interaction can provide valuable support for track condition evaluation. This paper investigates the use of equivalent conicity as a parameter for assessing the technical condition of railway track. Equivalent conicity reflects the combined influence of wheel and rail profiles, track gauge and rail inclination and is commonly applied in vehicle stability analyses. The paper outlines methods for determining equivalent conicity and discusses the limitations resulting from the nonlinear nature of wheel–rail contact, especially under wheel and rail wear. The relationship between equivalent conicity, vehicle stability and critical speed is analysed, with particular attention paid to the ambiguity of single conicity values used as assessment criteria. The study indicates that equivalent conicity, when treated as a track-related parameter using reference wheel profiles and supported by wear analyses, can contribute to a more comprehensive and safety-oriented evaluation of track condition.

1. Introduction

This publication discusses the use of equivalent conicity as a parameter for assessing the technical condition of a track. This value is commonly used in assessing wheel-rail interaction, for example during type approval tests of rail vehicles, as information needed for analyzing ride stability [4, 13]. Some railway infrastructure managers take the equivalent conicity parameter into account when assessing the condition of the track. Equivalent conicity is an important parameter in assessing operational safety, hence it is essential to correctly assess and interpret the results obtained.

2. Track condition assessment tools

The assessment of the operational capacity of railway infrastructure is largely based on indirect and direct measurements carried out using available technologies. The criteria for assessing selected infrastructure parameters are largely determined by the infrastructure manager. In addition to track system parameters related to safety and stability, the results of analyses of additional signals related to the dynamic response of the vehicle are also taken into account in the analyses [21]. Due to ongoing digitalisation and staffing problems occurring almost worldwide, modern tools and algorithms are increasingly being used to effectively assess the technical condition of the track. Analytical tools based on theoretical assumptions [12], actual measurement results [2, 6] and tools that take into account both sources of information are used. Most measurement vehicles that perform periodic measurements and form the basis for track condition assessment collect data on track condition by measuring basic geometric parameters, gauge geometry, profiles, defectoscopy and dynamic response. In addition, infrastructure managers decide to introduce additional functionalities related to the actual signal recorded on operational vehicles (e.g. vibration acceleration) and theoretical studies related to the assessment of driving safety based on the analysis of equivalent conicity.

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3. Determination of equivalent conicity

Equivalent conicity describes the difference between the lengths of the rolling radii of the wheels of a wheel set as a function of its lateral displacement (Δr(y) relationship). It is therefore an indicator that characterises the interaction between the vehicle and the track and links together variables such as wheel and rail profile geometry, track gauge and rail inclination. The equivalent conicity value for a wheel set lateral displacement of 3 mm is used to describe the interaction between the vehicle and the track in standard [13]. Equivalent conicity is based on linearisation or quasi-linearisation (linearisation for specific ranges of values) of the Δr(y) function. This approach is dictated by the need to use linear relationships in calculations concerning vehicle dynamics, including calculations of critical speed using the eigenvalue method. However, the mechanics of wheel-rail contact is a highly non-linear case, which is why the essence of equivalent conicity as a reliable and deterministic parameter for describing the dynamic behaviour of a vehicle on a track is still the subject of lively scientific debate [8, 14].

The essence of the concept of equivalent conicity depends on the type of wheel profile. For a wheel with a conical profile (straight cone), the difference in the rolling radii of the wheels in a wheel set is a linear function of the lateral displacement of the set. For this reason, the value of equivalent conicity is equal to the angle of inclination of the wheel's rolling surface [10]. The lack of a linear conical profile of actual (nominal and worn) profiles implies a non-linear nature of conicity. For this reason, there are different approaches to the mathematical determination of equivalent conicity.

The first approach defines equivalent conicity as the difference between the rolling radii of the inner (r_w) and outer (r_z) wheels of the wheel set, relative to twice its transverse displacement (Δy). This relationship is used to evaluate equivalent conicity and is described in Eq. (1) [5]. It should be noted that in the case of large non-linearities in the geometry of the wheel-rail contact, this method does not produce reliable results [5].

$$\lambda_{eq} = (r_z - r_w)/2\Delta y = \Delta r/2\Delta y \qquad (1)$$

Andersson et al. in [1] proposed improving the previously presented method (Equation 1) by calculating the average equivalent conicity for a specific number of transverse displacement values of the wheel set until the amplitude of the transverse displacement value is obtained. It is then proposed to map the resulting function using linear regression with the least squares method.

Polach in [14] presented methods for determining the equivalent conicity value by quasi-linearisation and three ways of obtaining its value by linearisation: harmonic, equivalent and using linear regression.

The determination of the equivalent conicity value in the case of a quasi-linear model of a wheel-rail system is based on the use of three parameters: the rolling radius of the wheel (R_w), the radius of the rail ($R_r$and the value of the contact angle (δ_0). The mathematical formula for this approach is presented in Equation (2).

$$\lambda_{eq} = \delta_0\left(1 + \frac{R_R}{R_w - R_r}\right) \qquad 2$$

The harmonic linearisation method aims to obtain the smallest possible mean square error between the expected values of the non-linear relationship Δr(y) and the values of the quasi-linear function, over an integration period equal to one wheel revolution. The mathematical notation for this method is presented in Equation (3).

$$\lambda_{eq} = \frac{1}{2\pi A}\int_0^{2\pi} \Delta r(A\sin\varphi)\sin\varphi\, d\varphi \qquad (3)$$

where: A – amplitude of sinusoidal motion, φ – angle.

Equivalent linearisation involves determining the wave length (L) of the wheel set and using the Kllingel formula to calculate the equivalent conicity value (Equation 4).

$$\lambda_{eq} = \left(\frac{2\pi^2}{L}\right)e_0 r_0 \qquad (4)$$

where: e_0 – half the pitch diameter of the rolling circles, r_0 – rolling radius of the wheel.

Another approach describing the essence of equivalent linearisation indicates that the equivalent conicity value for the actual profile determines the angle of inclination of the ideal conical profile for which the waviness length would be the same as in the case of the actual profile [4].

The linearisation of the Δr(y) relationship using linear regression is based on determining the regression coefficient (k) equal to the value 2λ_eq and is presented in Equation (5).

$$\lambda_{eq} = \frac{k}{2} \qquad (5)$$

In addition to analytical methods for determining the equivalent conicity value, attempts have been made in the literature to determine this value experimentally. In [11], an innovative method of monitoring the equivalent conicity value using Kalman filters is presented. The results obtained through experimentation were validated using results generated by MBS simulation. The validation confirmed that the new method is reliable and can be used to determine the equivalent conicity value for nominal and worn profiles under various friction conditions between the wheel and the rail. Another method for determining the equivalent conicity value can be optical measurements of a wheel-rail pair, which are coupled with calculation modules [3].

If the equivalent conicity value is high, the wavelength of the snake effect is shortened, resulting in an increase in transverse vibration frequency while driving. If the vehicle has not been adapted to reduce this type of vibration, we will experience intense corrugation, generating high forces between the wheel and the rail, leading to deterioration of the technical condition of the rolling stock and infrastructure, and ultimately causing derailment. As the driving speed increases, the equivalent conicity should be lower to prevent instability of movement [14].

4. Equivalent conicity in assessing wheel-rail interaction

Equivalent conicity is used primarily in linear analyses of rail vehicle stability, which is an important issue in terms of safety and comfort. Stability assessment involves determining the critical speed, i.e. the speed above which the vehicle will start to behave unsteadily on the track [15]. An increase in the equivalent conicity value reduces the critical speed, resulting in unstable vehicle movement at lower speeds. However, in the case of non-linear simulations, which also assume, among other things, forces from the track or the non-linearity of the vehicle's compliant and friction elements, equivalent conicity is not used. This is a significant limitation of the usefulness of this dynamic indicator. Furthermore, due to its linear nature, equivalent conicity is not always a reliable indicator of the dynamic properties of the vehicle-track system. Adopting a linear model of wheel-rail contact mechanics results in a lower critical speed value being obtained from dynamic calculations. An example from engineering practice is the case of the Danish railways, which intended to increase the speed of their vehicles. Despite the stable (safe) dynamic behaviour of the vehicle during the experiment and numerical calculations, the operator did not approve it due to the higher equivalent conicity value than that specified in the standards [8].

As can be seen from the information presented, equivalent conicity refers to a pair of wheels and rails with specific profiles, rather than to individual components. Therefore, during the inseparable wear and tear of wheels and rails in rail vehicles, their profiles change, and as a result, their mutual interaction, determined, among other things, by equivalent conicity [5, 18, 23]. This process is accompanied by the formation of a so-called false flange, which completely changes the characteristics of the equivalent conicity and, as a result, impairs the self-centring ability of the wheel sets and reduces the critical speed (with a frequent, simultaneous increase in the amplitude of oscillations).

In [23], it was observed that the more concave the shape of the equivalent conicity with respect to the transverse displacement of the wheel set, the greater the non-linearity of the wheel-rail contact characteristics. The concept of equivalent conicity is related to the linearisation of wheel-rail contact. As a result of this simplification, it is possible to describe an infinite number of wheel and rail profile pairs for a transverse displacement of the wheel set equal to 3 mm (Fig. 1) [16]. This issue was also highlighted in [14]. The vast majority of wheels moving on the track infrastructure are worn to some extent, resulting in non-linear wheel-rail contact. In view of the above, it seems reasonable to introduce additional (apart from equivalent conicity) quantities describing the contact. The concept of equivalent conicity as a decision parameter in the process of approving rail vehicles (in terms of running stability) is also contested by True in [22].

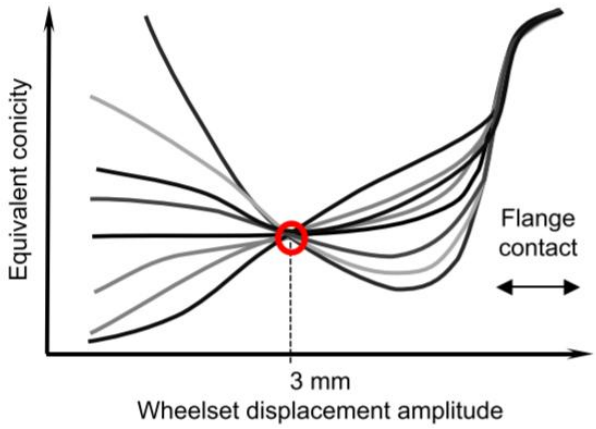

Fig. 1. The ambiguity of the equivalent conicity value, based on [16]

One of them is the nonlinearity coefficient λN, which, together with the equivalent conicity allowing for a quantitative assessment of instability (in relation to permissible values), will enable its nature to be determined. The authors determined the dynamic tendencies of wheel-rail pairs depending on the sign of the λN coefficient. Pairs with a positive λN value are characterised by a sudden transition from stable running to a limit cycle of wheel set oscillations with significant amplitudes. Pairs with negative values of the nonlinearity coefficient exhibit a lower critical speed, above which oscillations with a small amplitude increasing with driving speed occur. In addition, such pairs generate higher lateral forces and wheel set oscillation amplitudes for subcritical speeds. The nonlinearity coefficient is determined using formula (1), where yλ denotes the value of lateral displacement read from (2).

$$\lambda_N = \frac{\lambda(y_\lambda+1) - \lambda(y_\lambda-1)}{2} \tag{6}$$

$$y_\lambda = 3 \text{ mm}, \quad \text{if } l_z \geq 7 \text{ mm}$$
$$y_\lambda = \frac{l_z - 1}{2} \text{ mm}, \quad \text{if } 5 \text{ mm} \leq l_z < 7 \text{ mm}$$
$$y_\lambda = 2 \text{ mm}, \quad \text{if } l_z < 5 \text{ mm} \tag{7}$$

The equivalent conicity value increases with mileage, which is caused by the increasing conformity of the wheel and rail shapes and the expansion of the contact area (in the transverse direction). After a certain degree of wheel profile wear has been reached, these parameters assume an approximately constant value [16]. The same paper also describes two other parameters of wheel-rail contact: the contact concentration index (CCI) (calculated by averaging the wheel-rail contact concentration Cc in a normal distribution in the range –3σ–3σ of the lateral displacement of the wheel set with a specific standard deviation σ) and the contact bandwidth change rate – V_w (the quotient of the contact area width and the amplitude of lateral displacements of the wheel set in the track) [7]. Based on these parameters, it was concluded that wheel and rail pairs with low CCI and high V_w should have a stable rolling surface shape as wear progresses. However, in many cases, the new wheel profile differs significantly from the worn profile due to improper matching to the rolling surface of the rails.

Another phenomenon that disturbs the conicity process is the formation of false edges on the wheels of rolling stock. They are formed on the wheel profile on the outer side of the front plane as a result of uneven wear of the material, mainly in the central part of the rolling surface (Fig. 2).

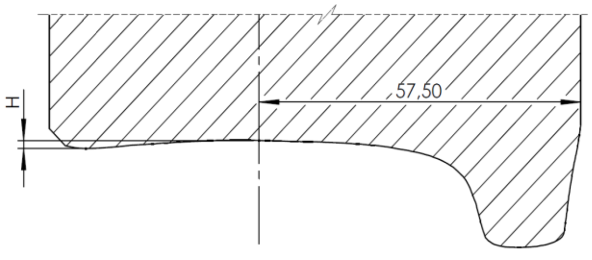

Fig. 2. False flange (H – height of false flange)

False flanges cause significant changes in the function of the rolling radius difference relative to the transverse displacement of the wheel on the rail, and as a result, they disrupt the kinematic guidance of wheel sets, even leading to derailment of vehicles [9]. This confirms the need to implement false spot height measurements in operational practice. In [17], based on experience gathered for freight trains in North America, a dimension of H = 3 mm was proposed as the limit for deciding on wheel reprofiling.

5. Equivalent conicity as a parameter for assessing the technical condition of the track

Equivalent conicity is a value characterising the interaction between a wheel and a rail. It cannot be determined for the rail or wheel alone. In order to isolate the assessment of the variability of equivalent conicity over time for a single element of a wheel-rail pair, reference profiles of wheels and rails are used. These are most often nominal profiles [16]. Due to the variability of this parameter with the wear of wheels and rails, worn reference profiles must also be taken into account. Between 2009 and 2013, a European project entitled DYNOTRAIN (Railway Vehicle Dynamics and Track Interactions. Total Regulatory Acceptance for the Interoperable Network) was carried out, the aim of which was to update the certification requirements for rail vehicles for interoperable transport in European Union member states in order to increase the efficiency of the approval process [14]. As part of the project, new equivalent conicity limit values for assessment track sections in the PN-EN 14363 standard [13] were developed. They refer to the track, so the use of reference wheel profiles was assumed. According to TSI Infra, equivalent conicity is also a track parameter [14].

As described earlier, the concept of equivalent conicity is related to the stability of rail vehicle movement. Considering that the value of equivalent conicity increases with rail profile wear, this leads to a reduction in the critical speed of the vehicle on a given section of the route.

In publication [15], Polach pointed out that equivalent conicity can, by inversion, describe both the wheel (association of the measured wheel profile with the nominal rail profile) and the track in the case of association of the measured track profile with the nominal rail wheel profile.

6. Summary

Due to non-linear contact, the equivalent conicity value given for a single value of transverse displacement of the wheel set provides only a fraction of the information about the interaction between the wheel and the rail.

The non-linearity of contact increases with the wear of both the wheel profile and the rail. This is particularly important for assessing ride stability, as the critical speed of the vehicle decreases for heavily worn wheel and rail profiles. The equivalent conicity value should be calculated for the current condition of the wheel running surface.

A single equivalent conicity value can describe many wheel-rail pairs with different dynamic behaviour, which means that the obtained conicity values may be exceeded and provide false information regarding the assessment of the interaction between the vehicle and the track, including the introduction of operating speed restrictions. Therefore, in order to verify possible operating scenarios and minimise risk, the analysis of equivalent conicity values should also be carried out for different degrees of wheel and rail wear. It is important to take into account the actual parameters of the rail and wheel profile when assessing the condition of the track, and to carry out simulations that take into account the worst-case scenarios: maximum wheel wear in the context of the current rail profile. The assessment of equivalent conicity can be part of a systematic assessment of the technical condition of the infrastructure.

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